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IDEOGRAPHIC COMPUTATION IN THE PROPOSITIONAL CALCULUS

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This paper will be concerned with an adaptation of Parry's trapezoid symbolism[1] to the solving of problems.

In Parry's notation '*p*_*q*', in particular, sets forth the relationship that obtains when *p* is true, whether *q* be true or false. The symbol may also be regarded as manifesting the matrix analysis of *p*. It is this aspect which offers promise for manipulation. We propose to write the symbol '_ ' for *p* itself. It is to be kept in mind, however, that '_ ' is the symbol of *p* when there are but two variables. How to proceed when there are more than two will be seen later.

The symbol for *q* will in like manner be ' _/'.

Let '_ ' and ' _/', then, be regarded as elemental symbols setting forth the matrix analysis of the variables. Suppose further that in transcribing from conventional to the trapezoid symbolism we substitute for the variables only, retaining the constants (antithetically to Parry's procedure). Thus '$p \vee q$' becomes, not '*p*_/*q*' as with Parry, but '_ ∨ _/'. Problem-solving consists of merging the trapezoidal symbols according to procedures specified by the constants. As symbols are merged, the constants are eliminated. The resultant symbol constitutes the solution.

The rules for merging symbols are derived from the elementary processes of forming truth-tables:

Disjunction of two symbols consists of superimposing them.

Conjunction consists of preserving only such sides as are common to both the conjoined symbols.

Negation of any symbol consists of writing a 'complementary' one, i.e., one in which just the sides not in the original are shown.

The symbol resulting from an implication consists of the negation of the symbol of the antecedent superimposed on the symbol of the consequent.[2]

The symbol of an equivalence consists of those sides appearing either twice or not at all in the merging symbols.

Non-equivalence (the exclusive *or*) consists of those sides that occur once only in the merging symbols.

In any of these merging operations, the mark 'x' will be treated as a 'blank.'

When the number of variables, *n*, is greater than 2, the new symbols for

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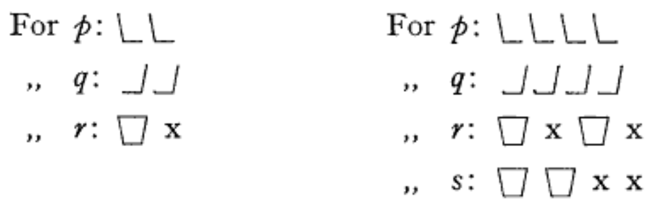
[1] As set forth in his paper in this Journal, vol. 19 (1953), pp. 161–168.

[2] I.e., '$p \supset q$' is treated as '$\bar{p} \vee q$'.

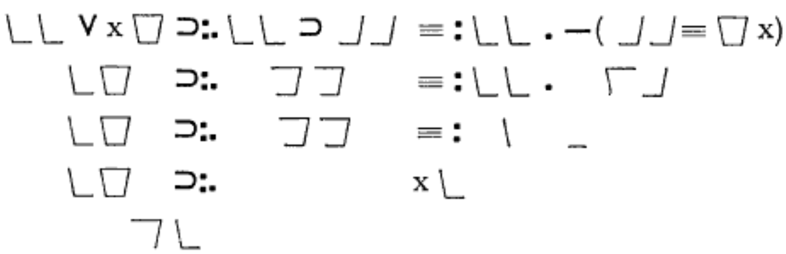
all but the last of them will be formed by doubling the symbols which were used for them when there were $n - 1$; and the symbol for the nth variable will consist of 2^{n-3} trapezoids followed by as many x's. Thus, where n is 3 and 4, we have:



Two devices serve to mitigate the difficulty of handling a large number of variables. One is an abbreviated form of notation. In dealing with six variables, for example, '16 └' is a space- and time-saving device for expressing p; similarly '8 ▽ x' represents r. For '$p \equiv s$' is written '4 └ └ ┐┐'. The other is the use of columnar arrangement when dealing with normal forms. This will be exemplified in the second problem to be solved. For both these devices I am indebted to Professor Parry.

We now pass to the exemplification of the technique. The first example is:

$$p \vee \bar{r} \supset :. p \supset q \equiv : p \,.\, -(q \equiv r)$$



The antecedent of this expression is seen to be the subaltern of the consequent when their reduced symbols are compared.[3] The final symbol informs us that the original function is a contingency. It manifests the truth conditions of the original (the Boolean expansion will have four elements). From it may be seen what the simplest equivalent of the original expression is, in this case '$\bar{p}r \vee p\bar{r}$' or '$r \equiv \bar{p}$'. (Such renderings will be explained later.)

The second example is a tautology with four variables, borrowed from Professor Quine's ***Methods of logic***.[4]

$$pq \vee p\bar{r} \vee \bar{p}r \vee \bar{p}s \vee \bar{q}r \vee \bar{r}\bar{s}.$$

The solution of this problem appears at the foot of the following table. Each trapezoid therein is the result of superimposing in one operation the several symbols directly over it.

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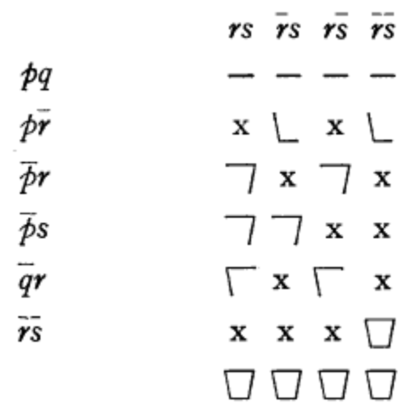
[3] One trapezoidal expression implies another if the antecedent can be contained in the consequent.

[4] Willard Van Orman Quine, ***Methods of logic***, (New York, 1950), p. 30.



That this solution is a tautology is signified by the presence of every possible side. There is no truth condition which fails of being true. The equivalent expression is '$p \vee \bar{p}$', or '$q \vee \bar{q}$', or '$r \vee \bar{r}$', or '$s \vee \bar{s}$'.

Headings like the above can be helpful in writing not only conjunctions of simple variables, but any p, q function to which is conjoined any number of the higher variables. The symbol for any such expression is comprised (1) of x's under those headings which contain, with opposite sign, any of its literals; and (2) of the symbol of the p, q function under the remaining headings.[5]

These headings also aid in rendering back into conventional form a given trapezoid symbol: Of course any symbol can be rendered by its component sides into a Boolean expansion: the present device allows for translating heading by heading into a disjunction, each member of which conjoins a heading to the p, q function symbolized thereunder. Even simpler renditions are sometimes possible by noting (1) that where the same sides occur under two headings which are alike save for one literal (generalizing: 2^x headings alike save for 2^{x-1} literals), those literals may be dropped in rendering those sides (Cf. lines 3 and 1 of the second example); and (2) that where the same sides occur under 2^x headings, related as noted, and the complementary sides appear under the remaining headings, all these may be rendered as an equivalence between the conjunction common to the related headings and the p, q function expressed by the sides thereunder (cf. the solution of the first example, which is a simple case of the rule here generalized).

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[5] Where the conjunction contains no explicit p, q function, it may be treated as conjoined to '$p \vee \bar{p}$' (cf. line 6 of the second example).